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Optimal distribution of a heat source within a thermopenetrator

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Abstract

Heat conduction within a heater of an arbitrary shape is investigated. A mathematical model is presented as a mixed boundary-value problem for the Poisson equation converted into a Fredholm boundary integral equation of the first kind which is solved numerically. A closed-form solution for the particular case of a rectangular heater is also found. Provided that the temperature and heat flux on the heater's boundary are given, the problem is treated as an inverse problem where the heat source distribution within the heater is the unknown function. The existence of the unique solution of this inverse problem is proved. Finally, the problem is solved numerically for a one-dimensional heat source. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Heat conduction; Integral equation; Heat source; Optimization; Inverse problem

1. Introduction

The analysis of the heat conduction within a heater is provided in this paper. This problem is treated as an inverse problem, where the heat source distribution within the heater is to be found. The following methods should be noted among the techniques developed for the solution of the inverse heat conduction problem (IHCP): function specification method, as proposed by Beck et al. [1], whole time domain regularization method [2], combined function specification and regularization method [3], mollification method proposed by Murio [4], and the boundary element approach for IHCP in application to a bi-dimensional transient numerical experiment that was implemented by Pasquetti and Le Niliot [5]. The above-mentioned studies deal with unsteady heat conduction problems and the major concern when attempting to computationally solve these problems has been with the automatic filtering of noisy data in the discrete thermocouple measurements. All measurement data errors, as well as numerical round-off errors, are amplified by the typical unsteady inverse heat conduction algorithms. Martin and Dulikravich [6] developed a non-iterative algorithm, based on the boundary element method (BEM), which can efficiently solve IHCPs which are governed by the Laplace equation in two-dimensional multiply connected domains. The results obtained in Ref. [6] for the unknown heat source distribution produced errors that were, at worst, in error by about 30%. The authors attributed this to the fact that the assembled BEM matrix had more unknowns than it had equations. However, the results were improved whenever internal temperature measurements were included in the analysis.

In the present paper, a two-dimensional model is proposed. The heater is assumed to be symmetrical relative to its central line with a symmetrical heat source distribution. The latter assumption allows us to consider only half of the domain, and this approach makes it possible for us to construct the non-traditional Green's function for the direct heat conduction problem with mixed boundary conditions. The integral equation obtained is much more convenient than the traditional one for the numerical computation of the heat flux on the working surface of the heater since it does not contain unknown temperatures. Moreover, the calculation of the Green's function derivative is not required for its solution, and this is also another advantage over the traditional approach quoted in Refs. [5,6].

Although the theoretical analysis provided in the present paper is related to the general steady IHCP with an unknown heat source, the numerical calculations for the particular case of one-dimensional heat source are performed. This is because the tendency of the heat source distribution along the central line is of major concern for the practical design

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